Stabilized mixed finite element modeling of unsaturated flow barrier and fractured porous media at finite strain

**WaiChing Sun**, Columbia University, wsun@columbia.edu

Many man-made and natural geological processes may lead to the inceptions and propagations of narrow zones in which significant inelastic deformation concentrates. Examples include shear bands, compaction bands, fractures and joints. Due to the size difference between the thickness of the localized zone and the rest of the deformable body, these geological features can be approximated as a strong discontinuous displacement field across interfaces. Depending on how microstructural attributes (e.g. porosity, tortuosity and pore size distribution) evolve, these localized geological features may become flow barriers or conduits. These changes in flow properties must be handled properly [1,2].

The objective of this study is to derive and implement computer models to capture these multi-physical processes via mixed localization finite element. Our formulation features an equal-order mixed finite element discretization in which an eight-node brick element is used to discretize both the displacement and pore pressure of the porous media [3,4]. The projection-based stabilization procedure is established by introducing an additional term in the weak form of the balance of mass equations that reads,

$$R^{stab}(v,p) = \sum_{K \in \Omega} \int_K (v - \Pi(v)) \tau (p - \Pi(p)) dV \tag{1}$$

where v is the trial function of the pore pressure, p is the interpolated pore pressure and Π is a projection operator that maps the original pore pressure field to an element-wise constant field.
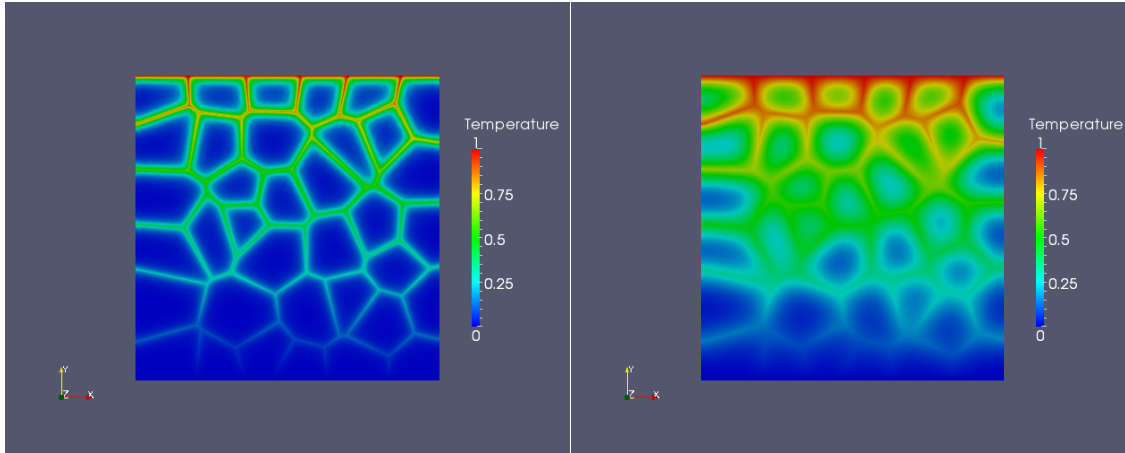
Figure 1. Temperature profile of material with thermal diffusivity in the bulk volume 3 orders lower than those in the grain boundaries.

By introducing a projection-based stabilization technique, the model is capable of producing stable solution with arbitrary time step size and drainage conditions. Meanwhile, the fully coupled diffusion-deformation mechanism of the localized zone is captured by a multi-physical variational localization element that admits displacement and pore pressure jumps. Inside the localization element, displacement and pore pressure are expressed in a curvilinear coordinate system that reads,

$$X(\xi^1, \xi^2, \xi) = X_M(\xi^1, \xi^2) + \xi n(\xi^1, \xi^2) \tag{2}$$

The gradient of the pore pressure and displacement are then described as functions of the pore pressure and displacement jump along the norma*l* direction the surface *[[p]] = p⁺ - p⁻, [[u]] = **u**⁺ - **u**⁻* and the co-variant base vectors which reads,

$$g_a = \frac{\partial X}{\partial \xi^a} = a_a + \xi \frac{\partial n}{\partial \xi^a} \quad ; \quad g_3 = n = \frac{a_1 \times a_2}{\| a_1 \times a_2 \|} \quad ; \quad a_a = \frac{\partial X_M}{\partial \xi^a} \quad ; \quad a = 1,2 \tag{3}$$

To demonstrate the robustness and versatility of the proposed models, we conducted a number of numerical simulations. The first example is shown in Figure 1 in which the thermal conduction of a rigid porous media with thermal conductivity at the fractured lines 3 orders higher than that of the bulk volume is simulated. This example is introduced to study the coupled diffusion process of local features and host matrix that have profoundly different conductivity. The second example is shown in Figure 2, in which the deformation-induced seepage of a porous media is simulated, before and after the sliding event took place. Due to the discontinuity in displacement, we formulate the hydro-mechanical coupled problem in finite deformation regime.
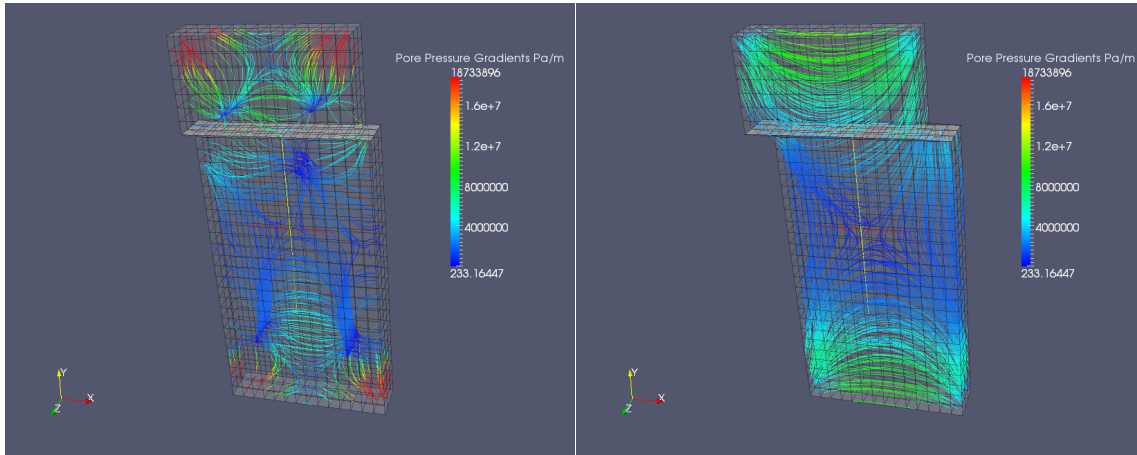

Figure 2. Simulated flow streamline across slip.

Acknowledgments

References
[1]   W.C. Sun, M.R. Kuhn and J.W.Rudnicki, A multiscale DEM-LBM analysis on permeability evolutions inside a dilatant shear band, *Acta Geotechnica,* 8(5):465-480, 2013.

[2]   W.C. Sun, J.E. Andrade, J.W. Rudnicki, A multiscale method for characterization of porous microstructures and their impact on macroscopic effective permeability, *International Journal for Numerical Methods in Engineering*, 88(12):1260-1279, 2011.

[3]   W.C. Sun, J.T. Ostien, A.G. Salinger, A stabilized assumed deformation gradient finite element formulation for strongly coupled poromechanical simulations at finite strain, *International Journal for Numerical and Analytical Methods in Geomechanics,* 37(16):2755-2788, 2013.

[4]   W.C. Sun, Q. Chen, J.T. Ostien, Modeling hydro-mechanical responses of strip and circular footings on saturated collapsible geomaterials, *Acta Geotechnica*, doi:10.1007/s11440-013-0276-x, 2013.